ME470: Uncertainty Quantification The Pre-homework
Mechanical Engineering
Stanford University March 31, 2011

This pre-homework is due Tuesday, April 12. Feel free to look up the answers in appropriate references, but please cite the reference. Each person should turn in his or her own write-up at the beginning of class the day it's due. No late homeworks will be accepted.

Problem 1.

Let s be a random variable uniformly distributed between 1 and 2. Compute E[s] and Var[s]. What is the second moment of s?

Solution:

Problem 2.

Let s be a standard normal random variable (i.e., Gaussian with zero mean and unit variance). Let $t = s^4$, and show that s and t are uncorrelated.

Solution:

Problem 3.

Again let s be a standard normal, and let $t = 1/(25 - s)$. What is E[t]? (Beware. This question is tricky.)

Solution:

Problem 4.

Let $\{p_0, \ldots, p_n\}$ be a set of polynomials that are orthogonal with respect to a weight function w on the interval [-1,1]. Show that $p_0, \ldots, p_n$ are linearly independent if no $p_i = 0$. (HINT: If $\alpha_0 p_0 + \cdots + \alpha_n p_n = 0$, multiply both sides by wp_i and integrate over $[-1, 1]$.)

Solution:

Problem 5.

Let A be a full rank matrix. Construct an orthogonal matrix Q such that QA is symmetric and positive definite. [HINT: Use the SVD.]

Solution:

Problem 6.

Let $x \in \mathbb{R}^n$. Determine constants c_1 and c_2 such that

$$c_1\|x\|_\infty \;\leq\; \|x\|_2 \;\leq\; c_2\|x\|_\infty, \tag{1}$$

where $\|\cdot\|_2$ is the standard 2-norm, and $\|\cdot\|_\infty$ is the standard infinity norm (or max norm).

Solution:

Problem 7.

Let A be an $m \times n$ full rank matrix with $m > n$. Consider the least-squares minimization problem

$$\underset{x}{\text{minimize}} \quad \|Ax - b\|_2$$

Write down the normal equations to compute the solution. Why is this a bad idea, and what's a better method? (For this question, I suggest reading Lecture 11 from Trefethen & Bau's *Numerical Linear Algebra*, which you can find on Google Books.)

Solution:

Problem 8.

Let A be an $m \times n$ full rank matrix with $m > n$, and let B be a $p \times n$ matrix. Consider the linearly constrained least-squares problem

$$\begin{array}{ll} \underset{x}{\text{minimize}} & \|Ax - b\|_2 \\ \text{subject to} & Bx = c \end{array}$$

Write down the KKT matrix for this problem.

Solution: