Multivariate Polynomials & Sparse Grids (Continued)

- Sparse polynomial representations are useful for constructing efficient surrogates of QOIs that depend on multiple parameters.
- See outline from last class.

PDEs with Random Inputs

- PDE models with random inputs are useful for representing physical systems with uncertainty.
- The random inputs add new dimensions to the solution, and discretization procedures must account for these added dimensions. Some call this discretizing the stochastic space (but they're really just input parameters).
- All of the polynomial techniques we have discussed apply the outputs of the PDE models with random inputs.

Goals for class

- Understand the construction of sparse grid collocation and the sparse pseudospectral method.
- See how PDE models with random inputs are characterized and discretized.
- Understand how the polynomial approximation methods are applied in this context.

Notation & Definitions

- Smolyak's formula:

$$A(l,d) = \sum_{l+1 \leq |\mathbf{i}| \leq l+d} (-1)^{d+l-|\mathbf{i}|} \binom{d-1}{l+d-|\mathbf{i}|} \mathcal{U}_{i_1} \otimes \cdots \otimes \mathcal{U}_{i_d}$$