Scattered Interpolation: Kriging and Radial Basis Functions

- When constructing surrogates for expensive function evaluations, we don't always have control over where we can evaluate the function.
- If the function evaluations are precomputed at specified points, how can we construct an accurate approximation?
- With surrogates on scattered points, we perform surrogate-based optimization.

Goals for class

- See how the kriging surrogate is constructed as a realization of a random field.
- See how radial basis approximations are constructed.
- Understand the connections between kriging and RBFs.

Notation & Definitions

- A spatially dependent random field: $f(x, \omega)$, where x is a spatial coordinate and ω is the random component.
- For a zero-mean field, the two-point covariance functions: $\langle f(x_1)f(x_2)\rangle = C(x_1, x_2) = g(|x_1 x_2|).$
- For points x_j in the spatial domain, the covariance matrix: $C_{ij} = \mathcal{C}(x_i, x_j)$.
- A radial basis function $\phi_i(x) = g(|x_i x|)$.
- Paul's favorite example of a radial basis function: $\phi_i(x) = \exp(-(x_i x)^2/\rho)$ with correlation length ρ .